QUESTION 2

2.1   Consider the geometric series: $4 + 2 + 1 + \dfrac{1}{2} + \dots$

        2.1.1   Does this series converge? Justify your answer.   (2)

        2.1.2   Calculate S_∞ .   (2)

2.2   Given: $\displaystyle\sum_{p=k}^{10} 3^{p-1} = 29\,520$. Calculate the value of k.

        (5)
        [9]

QUESTION 3

3.1   Consider the quadratic number pattern: $3 \,;\, 7 \,;\, 12 \,;\, \dots$

        3.1.1   Show that the general term of this number pattern is given by
$$T_n = \frac{1}{2}n^2 + \frac{5}{2}n.$$
        (3)

        3.1.2   What number must be added to T_{n-1} so that $T_n = 13\,527$?   (4)

3.2   Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

        3.2.1   Calculate the value of n if $T_n = 41$.   (3)

        3.2.2   A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.
                For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

            (a)   Write down the value of P_{41} .   (1)

            (b)   Calculate the value of the first term of the new arithmetic sequence.   (4)
                                        [15]

QUESTION 2

2.1   Given the arithmetic series: $7 + 12 + 17 + \ldots$

    2.1.1   Determine the value of T_{91} (3)

    2.1.2   Calculate S_{91} (2)

    2.1.3   Calculate the value of n for which $T_n = 517$ (3)

2.2   The following information is given about a quadratic number pattern:

$$T_1 = 3, \ T_2 - T_1 = 9 \ \text{ and } \ T_3 - T_2 = 21$$

    2.2.1   Show that $T_5 = 111$ (2)

    2.2.2   Show that the general term of the quadratic pattern is $T_n = 6n^2 - 9n + 6$ (3)

    2.2.3   Show that the pattern is increasing for all $n \in N$.   (3)

                                                                           [16]

QUESTION 3

3.1   Given the geometric series: $3 + 6 + 12 + \ldots$ to n terms.

    3.1.1   Write down the general term of this series.   (1)

    3.1.2   Calculate the value of k such that: $\displaystyle\sum_{p=1}^{k} \frac{3}{2}(2)^p = 98\,301$ (4)

3.2   A geometric sequence and an arithmetic sequence have the same first term.
- The common ratio of the geometric sequence is $\dfrac{1}{3}$
- The common difference of the arithmetic sequence is 3
- The sum of 22 terms of the arithmetic sequence is 734 more than the sum to infinity of the geometric sequence.

Calculate the value of the first term.   (5)

                                                                              [10]

QUESTION 2

2.1 Given the geometric series: $\dfrac{1}{5}+\dfrac{1}{15}+\dfrac{1}{45}+...$

    2.1.1 Is this a convergent geometric series? Justify your answer with the necessary calculations. (2)

    2.1.2 Calculate the sum to infinity of this series. (2)

2.2 An arithmetic and a geometric sequence are combined to form the pattern, which is given by: $P_n = x\,;\dfrac{1}{3}\,;2x\,;\dfrac{1}{9}\,;3x\,;\dfrac{1}{27}\,;...$

    2.2.1 Write down the next TWO terms of the pattern. (2)

    2.2.2 Determine the general term (T_n) for the odd terms of this pattern. Write down your answer in terms of x. (2)

    2.2.3 Calculate the value of P_{26}. (3)

    2.2.4 If $\displaystyle\sum_{n=1}^{21} P_n = 33{,}5$, determine the value of x. (6)

[17]

QUESTION 3

A quadratic sequence has the following properties:

- The second difference is 10.
- The first two terms are equal, i.e. $T_1 = T_2$.
- $T_1 + T_2 + T_3 = 28$

3.1 Show that the general term of the sequence is $T_n = 5n^2 - 15n + 16$. (6)

3.2 Is 216 a term in this sequence? Justify your answer with the necessary calculations. (3)

[9]

QUESTION 2

2.1   The first term of a geometric series is 14 and the 6th term is 448.

         2.1.1   Calculate the value of the constant ratio, r.   (2)

         2.1.2   Determine the number of consecutive terms that must be added to the first 6 terms of the series in order to obtain a sum of 114 674.   (4)

         2.1.3   If the first term of another series is 448 and the 6th term is 14, calculate the sum to infinity of the new series.   (3)

2.2   If $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$, determine the value of k.   (5)

     [14]

QUESTION 3

It is given that the general term of a quadratic number pattern is $T_n = n^2 + bn + 9$ and the first term of the first differences is 7.

3.1   Show that $b = 4$.   (2)

3.2   Determine the value of the 60th term of this number pattern.   (2)

3.3   Determine the general term for the sequence of first differences of the quadratic number pattern. Write your answer in the form $T_p = mp + q$.   (3)

3.4   Which TWO consecutive terms in the quadratic number pattern have a first difference of 157?   (3)

     [10]

QUESTION 2

2.1   The first term of an arithmetic sequence is -1 and the 7th term is 35.

       Determine:

          2.1.1   The common difference of the sequence   (2)

          2.1.2   The number of terms in the sequence if the last term of the sequence is 473   (3)

          2.1.3   The sum of the first 40 terms in this sequence   (2)

2.2 $75 ; 53 ; 35 ; 21 ; ...$ is a quadratic number pattern.

          2.2.1   Write down the FIFTH term of the number pattern.   (1)

          2.2.2   Determine the nth term of the number pattern.   (4)

          2.2.3   Determine the maximum value of the following number pattern:

$$-15 ; -\frac{53}{5} ; -7 ; -\frac{21}{5} ; ...$$

                                                                    (4)

                                                                    [16]